MEI STRUCTURED MATHEMATICS

CONCEPTS FOR ADVANCED MATHEMATICS, C2

Practice Paper C2-A

Additional materials: Answer booklet/paper
Graph paper
MEI Examination formulae and tables (MF12)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You **may** use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that you may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

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Section A (36 marks)

1 The gradient of a curve is given by $\dfrac{dy}{dx} = 2 - 5x$.

Find the equation of the curve given that it passes through the point (0, 1). [4]

2 **(i)** Write $\log_2 5 + \log_2 1.6$ as an integer. [2]

(ii) Solve the equation $2^x = 3$, giving your answer correct to 4 decimal places. [3]

3 On his 1st birthday, John was given £5 by his Uncle Fred. On each succeeding birthday, Uncle Fred gave a sum of money that was £3 more than the amount he gave on the last birthday.

(i) How much did Uncle Fred give John on his 8th birthday? [2]

(ii) On what birthday did the gift from Uncle Fred result in the total sum given on all birthdays exceeding £200? [4]

4 Find the equation of the tangent to the curve $y = x^3 + 2x - 7$ at the point where it cuts the y axis. [4]

5 **(i)** Express $2\sin^2\theta + 3\cos\theta$ as a quadratic function of $\cos\theta$. [1]

(ii) Hence solve the equation $2\sin^2\theta + 3\cos\theta = 3$, giving all values of θ correct to the nearest degree in the range $0^\circ \le \theta \le 360^\circ$. [4]

6 The angle of a sector of a circle is 2 radians and the length of the arc of the sector is 45cm.

Find

(i) the radius of the circle, [2]

(ii) the area of the sector. [2]

7 The first two terms of a geometric series are 5 and 4.

Find

(i) the sum of the first 10 terms, [2]

(ii) the sum to infinity. [1]

8   In the triangle ABC, AB = 5 cm, AC = 6 cm and angle BAC = 110°.

Find the length of the side BC.   [5]

Section B (36 marks)

9   The equation of a curve is given by $y = (x - 1)^2(x + 2)$.

    (i)   Write $(x - 1)^2(x + 2)$ in the form $x^3 + px^2 + qx + r$ where p, q and r are to be determined.   [2]

    (ii)   Show that the curve $y = (x - 1)^2(x + 2)$ has a maximum point when $x = -1$ and find the coordinates of the minimum point.   [7]

    (iii)   Sketch the curve $y = (x - 1)^2(x + 2)$.   [1]

    (iv)   For what values of k does $(x - 1)^2(x + 2) = k$ have exactly one root.   [3]

10   A function $y = f(x)$ may be modelled by the equation $y = ax^b$.

    (i)   Show why, if this is so, then plotting log$y$ against log$x$ will produce a straight line graph. Explain how a and b may be determined experimentally from the graph.   [3]

    (ii)   Values of x and y are given below. By plotting a graph of log$y$ against log$x$, show that the model above is appropriate for this set of data and find values of a and b given that a is an integer and b can be written as a fraction with a denominator less than 10.   [7]

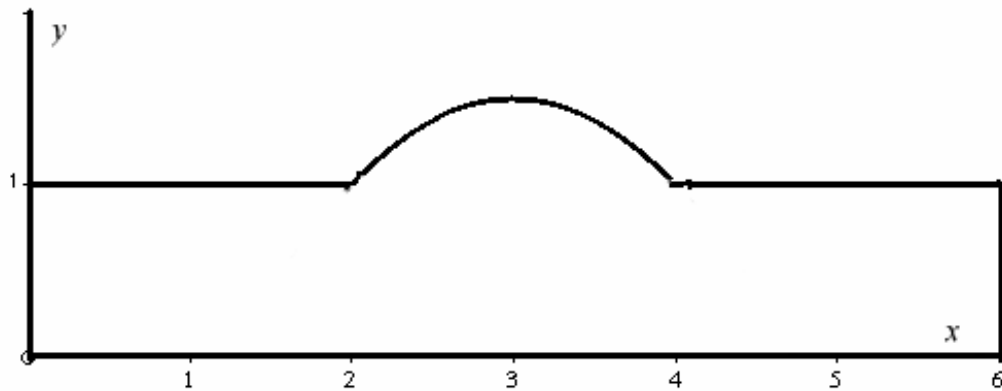| x	2	3	4	5	6
y	4.6	5.0	5.3	5.5	5.7

    (iii)   Use your formula from part **(ii)** to estimate the value of y when $x = 2.8$.   [2]

11   The cross-section of a brick wall built on horizontal ground is given, for $0 \le x \le 6$, by the following function

$$0 \le x \le 2 \quad y = 1$$

$$2 \le x \le 4 \quad y = -\frac{1}{2}x^2 + 3x - 3$$

$$4 \le x \le 6 \quad y = 1$$

Units are metres.

(i)   Show that the highest point on the wall is 1.5 metres above the ground.   [4]

(ii)   Find the area of the cross-section of the wall.   [7]